Suppressed Diffusion of Drag-Reducing Polymer in a Turbulent Boundary Layer

JIN WU*
Hydronautics Incorporated, Laurel, Md.

The concentration of drag-reducing polymer ejected from a thin wall slot into a developing turbulent boundary layer (external flow) was measured with a laser-phototransistor unit. Experiments included tests of ejecting polymer solutions of various concentrations at various ejection rates. The polymer solution was colored with dye and the additive concentration within the boundary layer was deduced from the dye concentration, which attenuated the laser light received by the phototransistor. The results indicate a suppressed turbulent diffusion of polymer solution.

Introduction

IT is well known[1] that a large drag reduction can be obtained by adding extremely small quantities of long-chain polymers to water. Although there is still missing a generally accepted mechanism[2] to explain this Toms' phenomenon,[3] it is commonly considered[2,4,5] that only additives in the wall region are responsible for drag reduction. Over a smooth surface, the wall region consists of the viscous sublayer and its immediate neighborhood, the buffer layer. Pursuing this line, experiments[5,6] have been conducted involving the introduction of polymer solutions from the wall into a turbulent pure-water boundary layer.

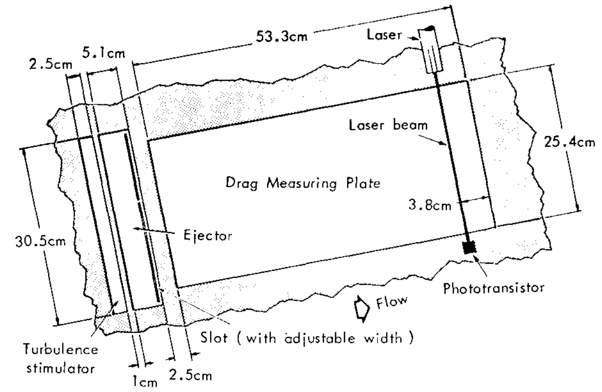
Since the ejected polymer solution is continuously mixed with turbulent water within the boundary layer, the drag reduction is largely determined by the diffusion rate of polymer solutions from the wall region. The dilution rate of polymer solutions in a turbulent boundary layer has been estimated[7,8] by assuming that the turbulent diffusion is not affected by the presence of polymers. However, such an analogy does not seem consistent with the observations[9,10] on suppression of free turbulence in polymer solutions. The purpose of the present experiment is to investigate possible suppressed diffusion of polymer solutions in a turbulent boundary layer.

Experiment

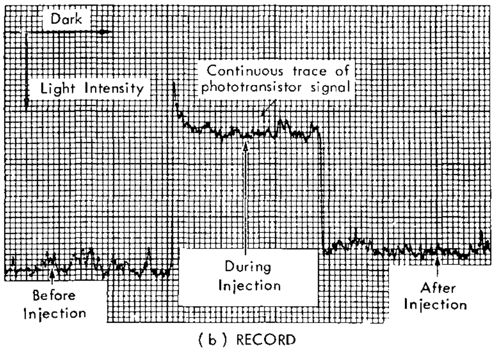
The experiment was performed in a circulating water channel, where drag-reduction studies[6,11] involving the use of uniform polymer-solutions and the ejection of polymer solutions into a pure-water boundary layer have been conducted. The channel has a closed test section 112 cm long, 38 cm wide and 19 cm deep. A part of the cover plate at the test section, 25.4 cm wide and 53.3 cm long, is cut from the rest with a clearance of 0.25 mm (1/100 in.) along four sides; see Fig. 1a. The resistance of this (floating) portion of the cover plate is measured with a strain-gage.

A laser (light source) was set in front of the test section and a phototransistor (light receiver) was placed on the back side; see Fig. 1a. The center of the laser beam, transversely across the channel and parallel to the plate, is located vertically 1.4 mm beneath the plate and longitudinally 3.8 cm upstream from the trailing edge of the floating plate. The laser and the phototransistor unit were calibrated with water of given dye concentration circulating in the channel. The light intensity, attenuated by dye and received by the phototransistor, can therefore be related to the dye concentration. For turbulent diffusion of very short time intervals, the additive concentration should be proportional to the dye concentration.

The experiments, like earlier drag-reduction studies,[6] were conducted at a constant channel velocity of 2.62 m/sec (8 fps). The ejected fluid consists of aqueous solutions of poly-(ethylene oxide) additive (Polyox WSR-301) of various concentrations. The polymer solution, colored by a given amount of dye, was introduced at different discharges from a pressurized reservoir through an inclined-slot ejector into the turbulent boundary layer developed along the cover plate. Near tangential ejection is achieved with a slot, 0.56 mm opening, at an inclination of 7° from the floating plate. During the experiment, the output of the phototransistor was continuously recorded before, during and after the ejection. A sample output is shown in Fig. 1b.



(a) ARRANGEMENT



(b) RECORD

Fig. 1 General arrangement and sample record.

Received June 10, 1971; revision received August 9, 1971.

This work was supported by the Office of Naval Research under Contract N00014-71-C-0063, NR 062-325. The author is grateful to N. Lewis for conducting the experiment and to the reviewers for their valuable comments.

Index category: Hydrodynamics.

* Principal Research Scientist. Member AIAA.

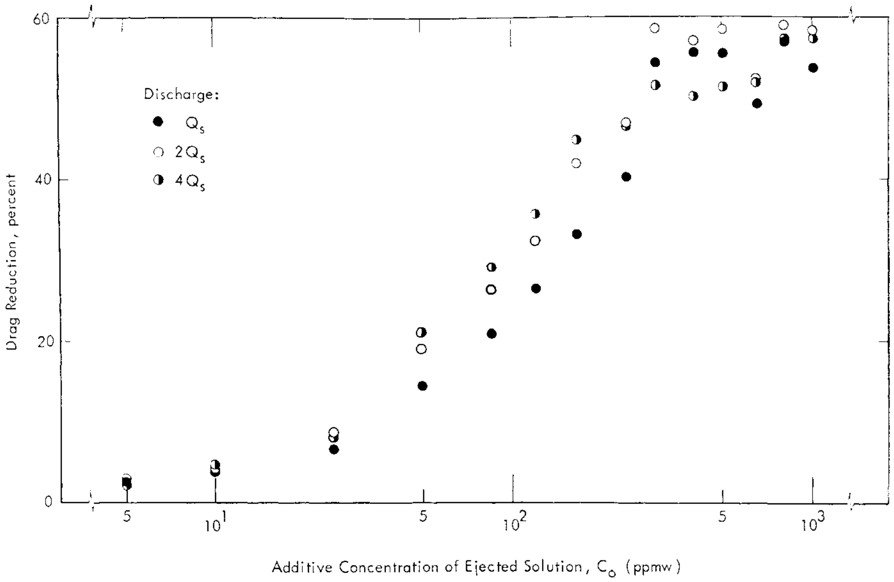


Fig. 2 Drag reduction by ejecting polymer solution into a pure-water boundary layer.

Description

Drag Reduction with Polymer Solutions Ejected from Slot

The drag reduction under identical experimental conditions was systematically measured earlier[6] and only verified occasionally during the present experiment. Portions of the results relevant to the present discussion are reproduced in Fig. 2: drag reduction obtained with polymer solutions of different concentrations ejected at rates equivalent to various multiples of the viscous-sublayer discharge (Q_s). The drag reduction is indicated by the ratio between the reduced drag and the turbulent frictional resistance. The viscous-sublayer discharge[6] is the flow rate within a layer having a nominal thickness of 11.6 $\nu/\bar{u}_*$, where ν is the kinematic viscosity of water and $\bar{u}_*$ is the mean friction velocity of the floating plate without additive ejection.

The features of the results have been discussed elsewhere.[6] It suffices for present discussion to point out that the drag reduction data shown in Fig. 2 reach a plateau at an additive concentration of 300 ppmw (parts per million by weight) for all three ejection rates.

Dimensions of Momentum and Diffusion Boundary Layers

Since the boundary layer along the channel roof starts to develop ahead of the floating plate, the average resistance coefficient of the plate was found smaller than the conventional value,[12] $0.074/Re^{1/5}$, (Re is the Reynolds number defined with the length of the plate). In order to find the boundary-layer thickness at the test station with the laser beam, it is then necessary to find the effective origin of the developing boundary layer by integrating the local resistance coefficient[12] C_f over the total length of the floating plate

$$C_f = 0.059/Re_{x'}^{1/5} \qquad (1)$$

and comparing the integrated resistance coefficient with the measured value, where $Re_{x'} = Ux'/\nu$ (U is the channel velocity and x' is the distance from the effective origin). In this way the effective origin was found to be 11.7 cm upstream from the leading edge of the floating plate.

The boundary-layer thickness δ at various distances from the effective origin, and the thicknesses of the viscous sublayer δ_s and of the buffer layer δ_b can be determined from

$$\delta u_*/x' = 0.377/Re_{x'}^{1/5}; \quad \delta_s u_*/\nu = 11.6; \quad \delta_b u_*/\nu = 30 \qquad (2)$$

and are shown in the Fig. 3, wherein u_* is the local shear velocity. It is noted that these are the dimensions for the pure-water boundary layer; these dimensions, especially the viscous-sublayer thickness, should be changed with the presence of polymers.

Mass diffusion from a two-dimensional source with a turbulent boundary layer was studied by Poreh and Cermak.[13] Their results indicate that the diffusion boundary layer can be divided longitudinally into four zones: 1) initial zone—the diffusion layer is submerged in the viscous sublayer. The thickness of the diffusion layer is the normal distance from the wall at which the mass concentration is half of that at the wall; 2) intermediate zone—the diffusion layer is submerged in the boundary layer but is thicker than the sublayer. The thickness of the diffusion layer $\delta_d = 0.076\, x^{0.8}$, where x is the distance from the origin of the diffusion boundary layer and the units are in cm; 3) transition zone—the ambient stream starts to influence the growth of the diffusion layer; and 4) final zone—the diffusion of mass extends beyond the boundary layer in this zone. The thickness of the diffusion layer, δ_d, continues to be a constant fraction of the boundary-layer thickness, $\delta_d/\delta = 0.64$. The locations of various diffusion zones are shown in Fig. 3. The test section with the laser beam is shown well within the final zone.

The intensity of the laser beam varies radially as the Gaussian distribution. The beam diameter is 1.4 mm, which is the distance between opposite radial points where the intensity is $1/e^2$ (0.135) of that at the beam center. The light is essentially a collimated beam with a 0.6-mrad divergence. The location of the beam center as well as the distribution of the light intensity normal to the plate are shown in Fig. 3. The skewed distribution is due to the use of logarithmical scale for the normal distance. The diameter at which the light intensity is half of that at the beam center is 0.82 mm, and may be considered as the effective sensing range, 76.2% of the total illumination falls within this diameter.

Wall Concentration and Concentration Profiles in Final Zone

According to Poreh and Cermak,[13] very large concentration gradients in the initial zone made it impossible to obtain reliable data. The wall concentrations C_w in the intermediate and the final zones were found to follow, respectively,

$$C_w = 26.2G\, x^{-0.9}/U; \quad C_w = (G/0.55)/U\delta \qquad (3)$$

where U is the ambient velocity (cm/sec) and G is the ejection rate (cc/sec/cm). The wall concentration is less certain in the transition zone. For an ejection rate twice than the viscous-sublayer discharge, the longitudinal variation of wall concentration calculated from empirical formulas, Eq. (3), is shown in Fig. 3.

A rather complicated expression was given by Poreh and Cermak[13] for the concentration profile normal to the wall in the final zone. Later, Morkovin[14] reanalyzed their data and

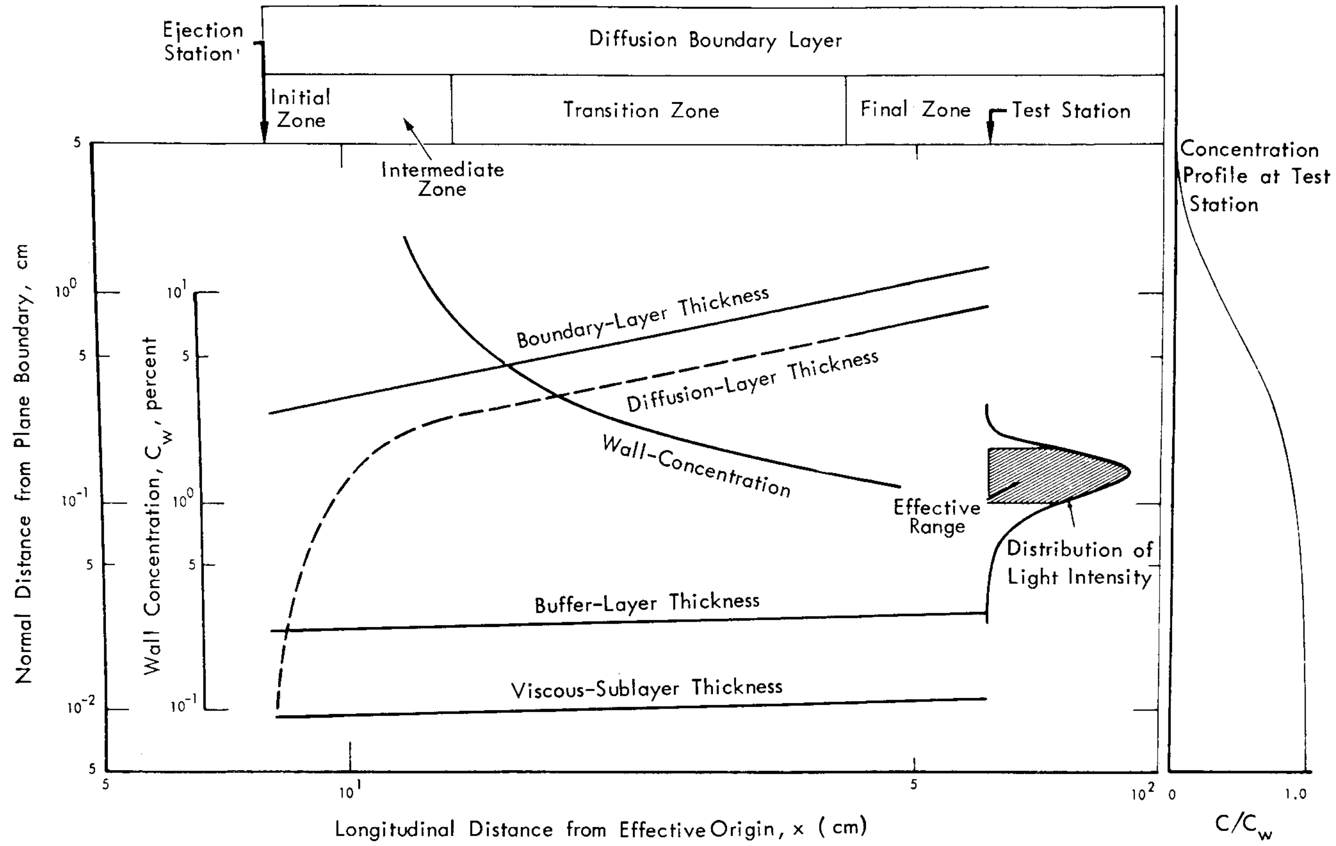


Fig. 3 Physical dimensions and wall-concentration distribution for pure-water boundary layer.

provided the following profile,

$$C/C_w = 1/\exp[0.693(y/\delta_d)^{2.15}] \tag{4}$$

where y is the normal distance from the wall; see Fig. 3. The concentration within the buffer layer but outside the viscous sublayer, where additives for drag reduction are operational, is seen to have essentially the wall concentration. The ideal location of the laser beam should be within this region. However, due to the finite diameter of the present laser it was impossible to place the beam any closer to the wall than that shown in Fig. 3. Consequently, for ejection of water the concentration measured by the laser is about 85% of the wall concentration.

Results

The step-like variation of phototransistor outputs, shown in Fig. 1b, displays distinctly the dye concentrations of the fluid for periods before, during and after ejection. From the deflection of the signal from dark, the concentrations of the fluid for three periods can be determined. Subtracting the dye concentration measured for the period before ejection from that during ejection, we obtained the added dye concentration due to the ejection. The accumulated dye concentration in the channel was kept at a very low level. The experiment was also benefited by a large volume of water in the channel. A flat rather than a rising trace was obtained during the ejection period; see the sample record shown in Fig. 1b.

The added dye concentration as a fraction, s, of the original dye concentration of the ejected fluid is plotted vs the ejection discharge in the upper half of Fig. 4. This fraction is proportional to the ratio between the added and the ejected polymer concentrations. The data are separately plotted according to the concentration of the ejected polymer solution. A continuous curve was then fitted through the data on the basis of the least square principle to relate the added polymer concentration to the ejection rate. From the fitted curve, the added concentration of the ejected polymer solutions at different ejection rates: Q_s, $2Q_s$, $3Q_s$ and $4Q_s$ can be obtained and are plotted in the lower half of Fig. 4. The discharge Q_s is again the flow rate within the aqueous viscous sublayer, discussed in an earlier section.

Discussion

General Assumptions for Estimating Diffusion of Polymers in a Turbulent Boundary Layer

The velocity distribution within the inner boundary layer is generally considered to be modified by polymers. The following correlation was presented by Meyer[15]

$$u/u_* = 2.5\ln(yu_*/\nu) + 5.5 + \Delta B; \quad \Delta B = \alpha\ln(u_*/u_{*c}) \tag{5}$$

where ΔB is the shift of logarithmic profile as a result of thickening the viscous sublayer, α is a numerical constant, and u_{*c} is the critical shear velocity for initiating drag reduction. Both α and u_{*c} are characteristics of the polymer solution. For dilute polymer solutions, u_{*c} seems to be independent of polymer concentrations, and α increases very slightly with the concentration. The outer law of the boundary layer is believed not to be affected by polymers.

The turbulent diffusion rate within the boundary layer is considered[16] to be proportional to the local shear velocity u_* or

$$d\bar{z}/dt = bu_* \tag{6}$$

where $\bar{z}$ is the mean normal elevation of the tracer from the wall, t is time and b is a proportionality constant. This constant has been considered to be the same in polymer solution and in water. As discussed in later sections, the last consideration should be re-evaluated.

Effects of Viscous Sublayer on Measurement of Concentration

Although the turbulent fluctuation is now believed to penetrate into the viscous sublayer, the mass diffusivity within the sublayer is still much smaller than that outside the sublayer.[17] Since the sublayer is extremely thin at high Reynolds

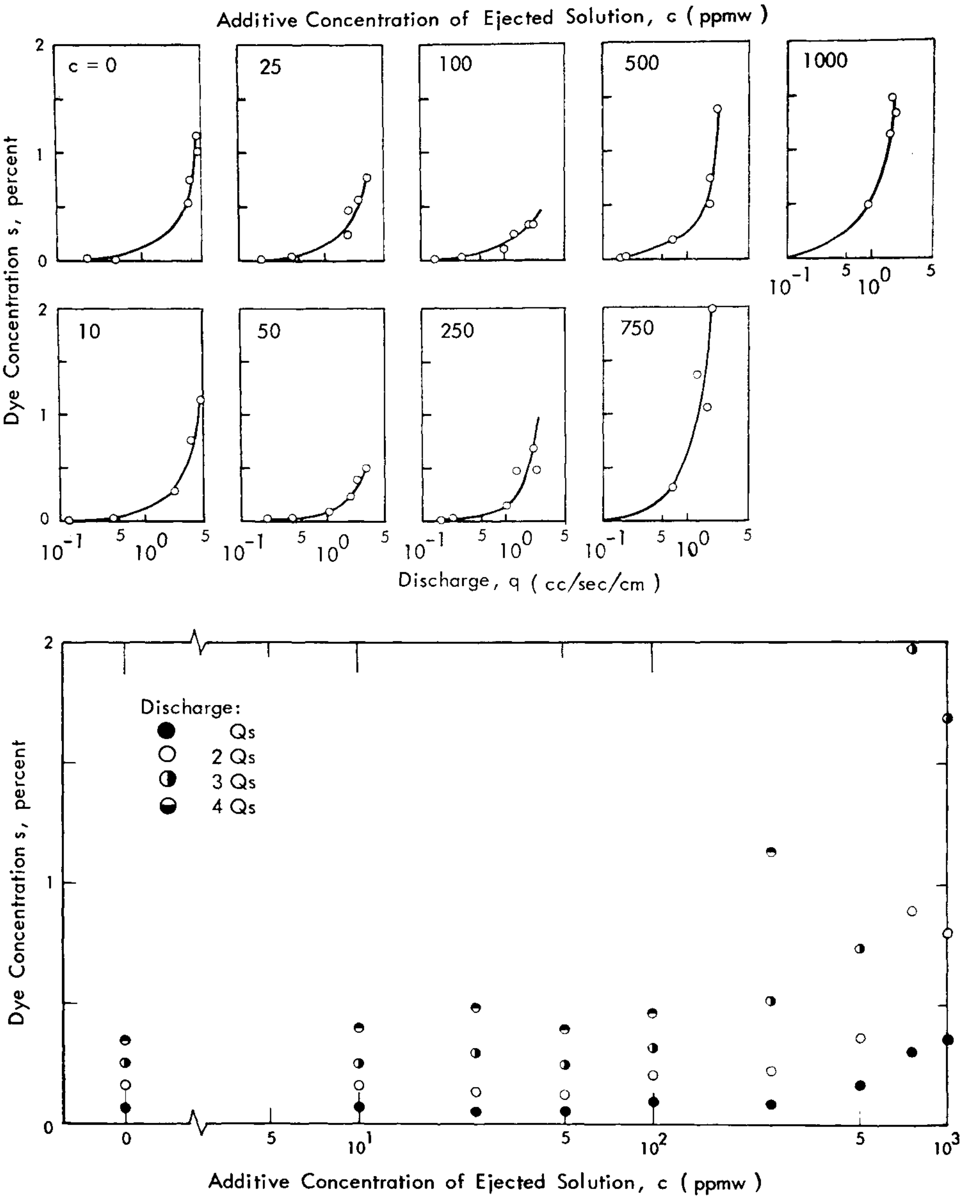


Fig. 4 Dye concentration measured at trailing edge of the plate.

numbers, its effect on diffusion of material ejected from the wall is generally neglected. However, when the ejection rate is comparable with the flow rate within the sublayer, a large portion of the tracer may be entrapped initially within the sublayer. This effect, of course, becomes less important at locations farther away from the ejection station. The influence of the viscous sublayer, nevertheless, exists even for very large distances; its cause then is not the initial trapping of the tracer but the slow exchange of tracer between the flow inside and outside the sublayer.[17]

It is well known[4,15] that polymers thicken the viscous sublayer

$$u_*\delta_s/\nu = a > 11.6 \qquad (7)$$

The amount of thickening, as shown in Eq. (5), is related to the shear velocity and polymer concentration. For cases with the same shear velocity, corresponding to the plateau portion of the drag-reduction data shown in Fig. 2, the test ejecting more concentrated solution probably has a thicker sublayer; because α, increasing slightly with polymer concentration, is greater. It can be shown that the flow rate within the sublayer is $a^2\nu/2$. Consequently, even a slight thickening of the sublayer increases appreciably the sublayer discharge.

Taken together the foregoing discussion, it seems that a greater fraction of the ejected tracer is entrapped within the sublayer for tests ejecting more concentrated solutions. Therefore, the turbulent mixing, or the dilution of polymers, outside of the sublayer, takes place with a smaller fraction of the ejected tracer. Consequently, for ejections with more concentrated polymer solutions, the dilution due to turbulent mixing should be less than that indicated by the dye concentrations, shown in Fig. 4 and expressed as the ratio between the measured and the original concentrations of the ejected solution. Because this dilution ratio is seen in Fig. 4 to be smaller at a lower-ejection rate, while a greater fraction of the ejected solution is entrapped within the viscous sublayer for ejections with more concentrated solutions and therefore results, effectively speaking, a lower-ejection rate.

Suppressed Diffusion of Drag-Reducing Polymers in a Turbulent Boundary Layer

The polymer concentrations measured with ejecting polymer solutions of various concentrations at various multiples of viscous-sublayer discharge are shown in the lower-half of Fig. 4. According to Poreh and Cermak,[13] for the ejection rate $2Q_s$ (shown in Fig. 3) the concentration measured by the laser at the test station should be about 0.6% of the original tracer concentration, which is more than twice the present measured value with pure-water ejection. This could be due to various reasons, such as different ways of ejecting the tracer, the initial trapping of the tracer within the viscous sublayer and the uncertainty of the displacement thickness of the sensor. (The dye concentration measured by the laser, spanning over a concentration profile, may not be the actual concentration at the geometric center of the beam). It is attempted here not to resolve this discrepancy but to compare the present measured values for different ejection conditions.

For ejections of polymer solutions of the same concentration, the measured dye concentration, as shown in the lower half of Fig. 4, increases faster than a linear proportion with the ejection rate. This is in favor of earlier discussion on the initial trapping of the ejected tracer within the sublayer, because the trapped tracer is a smaller fraction of the ejected tracer at higher ejection rates.

The data, shown in the lower half of Fig. 4, illustrate that there is very little change of turbulent diffusion with polymer concentration when the concentration of the ejected solution is less than 100 ppmw; that the dye concentration increases with the polymer concentration for ejected solutions having their concentration greater than 100 ppmw and increases very rapidly for ejected solutions having their concentration greater than 300 ppmw. The last portion of the data is the most interesting result of the present study, since, as shown in Fig. 2, the drag-reduction data for these concentrations already reach a plateau, while the laser-measured concentration increases drastically with the polymer concentration of the ejected solution. In other words, the drag-reduction data indicates that the average shear velocity is about the same, while the dye-concentration data indicates that the turbulent diffusion is much less effective. Moreover, the actual turbulent diffusion rate is probably even less than that indicated in Fig. 4, if, as discussed earlier, the initial trapping of the tracer is considered to be more severe at higher polymer concentrations. In summary, the present results provide evidence of a suppressed diffusion of drag-reducing polymers within a turbulent boundary layer.

Although there may be some disagreement between the present concentration measurements and those reported earlier by Poreh and Cermak,[13] the shape of the concentration profile reported by them is not likely to be changed. Their measured profiles correlated by Morkovin,[14] were recently verified by Fabula and Burns.[8] With water ejection, the tracer concentration measured by the laser, according to Eq. (4), is about 85% of the wall concentration. The thickening of the viscous sublayer with additive ejection will make the measured concentration even closer to the wall concentration. The portion of the data shown in the lower half of Fig. 4, for dilute polymer concentrations (less than 100 ppmw) should be very close to the pure-water ejection. The wall concentration for these cases should be slightly greater than the measured value. The higher dye concentration measured with ejection of more concentrated polymer solution (greater than 100 ppmw) is undoubtedly due to the thinning of the diffusion layer, because the laser-measured concentration in the latter cases (greater than 100 ppmw) is already greater than the wall concentration for the previous cases (less than 100 ppmw). Furthermore, the wall concentration for the latter cases (greater than 100 ppmw) should be greater than the laser-measured value. The thickness of the diffusion layer must be reduced to compensate for the increase of wall concentration.

Conclusion

The results seem to indicate a suppressed diffusion of drag-reducing polymer within a turbulent boundary layer. For practical use of polymers, the results suggest a re-evaluation of some assumptions involved in estimating polymer requirement for external flows. For studying polymer influence on turbulence, the results suggest a possible change of structure of large eddies; the latter is believed to govern the turbulent diffusion. The present study is obviously a limited one, far more investigations, especially detailed measurements of concentration profiles, are needed to clarify these points. Nevertheless, the present note will serve its purpose, if more attention is attracted to these problems.

References

[1] Hoyt, J. W. and Fabula, A. G., "The Effects of Additive on Fluid Friction," *Proceedings of the Fifth Symposium on Naval Hydrodynamics*, Office of Naval Research, 1964, pp. 947–974.

[2] Lumley, J. L., "Drag Reduction by Additives," *Annual Review of Fluid Mechanics*, edited by W. R. Sears, Vol. 1, Annual Review Inc., Calif. 1969, pp. 367–384.

[3] Toms, B. A., "Some Observations on the Flow of Linear Polymer Solutions Through Straight Tubes at Large Reynolds Number," *Proceedings of the International Rheological Congress*, North Holland Publishing Co., Amsterdam, Scheveningen, Holland, 1948, pp. 135–141.

[4] Tulin, M. P., "Hydrodynamic Aspects of Macromolecular Solutions," *Proceedings of the Sixth Symposium on Naval Hydrodynamics*, Office of Naval Research, 1966, pp. 3–18.

[5] Wells, C. S. and Spangler, J. G., "Injection of a Drag-Reducing Fluid into Turbulent Pipe Flow of a Newtonian Fluid," *Physics of Fluids*, Vol. 10, No. 9, 1967, pp. 1890–1894.

[6] Wu, J. and Tulin, M. P., "Drag Reduction by Ejecting Additive Solutions into a Pure-Water Boundary Layer," TR 353-7, 1970, Hydronautics, Inc., Laurel, Md.

[7] Granville, P., "The Frictionless Resistance and Velocity Similarity Laws of Drag-Reducing Polymer Solution," *Journal of Ship Research*, Vol. 12, No. 3, 1968, pp. 201–212.

[8] Fabula, A. G. and Burns, T. J., "Dilution in a Turbulent Boundary Layer with Polymeric Friction Reduction," NUC TP 171, 1970, Naval Undersea Research and Development Center, San Diego, Calif.

[9] Gadd, G. E., "Reduction of Turbulent Friction in Liquids by Dissolved Additives," *Nature*, Vol. 212, November 1966, pp. 874–877.

[10] Wu, J., "An Experiment on Turbulent Diffusion in Additive Solutions," *AIChE Journal*, to be published.

[11] Wu, J., "Drag Reduction in External Flows of Additive Solutions," *Viscous Drag Reduction*, edited by C. S. Wells, Plenum Press, New York, 1969, pp. 331–350.

[12] Rouse, H., "*Elementary Mechanics of Fluids*," Wiley, New York, 1957, Sect. 33.

[13] Poreh, M. and Cermak, J. E., "Study of Diffusion from a Line Source in a Turbulent Boundary Layer," *International Journal of Heat and Mass Transfer*, Vol. 7, No. 10, 1964, pp. 1083–1095.

[14] Morkovin, M. V., "On Eddy Diffusivity, Quasi-Similarity, and Diffusion Experiments in Turbulent Boundary Layer," *International Journal of Heat and Mass Transfer*, Vol. 8, No. 1, 1965, pp. 129–145.

[15] Meyer, W. A., "A Correlation of the Frictional Characteristics for Turbulent Flow of Dilute Non-Newtonian Fluids in Pipes," *AIChE Journal*, Vol. 12, No. 3, 1966, pp. 522–525.

[16] Cermak, J. E., "Lagrangian Similarity Hypothesis Applied to Diffusion in Turbulent Shear Flow," *Journal of Fluid Mechanics*, Vol. 15, Pt. 1, 1963, pp. 49–64.

[17] Yen, C. L., "Effects of Viscous Sublayer on Dispersion," *Journal of Sanitary Engineering Division, ASCE*, Vol. 95, SA6, 1969, pp. 1105–1115.